NETWORK FOLDING STRATEGIES FOR CONCURRENT ELECTROMAGNETIC FIELD MAPPING

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ABSTRACT

A number of concurrent algorithms for dynamic field mapping based on the two-dimensional lumped circuit analogies of Maxwell's equations are presented. Large networks of lumped equivalent circuits are mapped onto arrays of transputers to provide a computational advantage over classical sequential techniques. Different network folding and unfolding strategies are proposed to solve these large networks. Diakoptic methodologies in concurrent form are used throughout. The method presented is general and can be applied to any orthogonal coordinate system with non uniform elemental quantization and boundary conditions placed at infinity. Results are presented for a rectangular waveguide problem.

INTRODUCTION

A dynamic method based on the lumped circuit analogies that exist for Maxwell's equations in two-dimensions [1] is used to map the electromagnetic field. It is well known that a given medium can be represented by a lumped equivalent network, whose Kirchoff's equations are analogs of Maxwell's equations in finite difference form. Kron's original formulation [1] is completely general making it attractive for a broad variety of complex problems involving non uniform grid quantization and/or non standard coordinate systems. This paper presents the mapping of the lumped equivalent circuit of a rectangular waveguide propagating TE/TM modes. Throughout, the diakoptics [2] method is applied to manipulate the network in a piecewise fashion. This offers the advantage that, since subnetworks are independent they can be solved systematically using concurrent methods. Different strategies are proposed to manipulate these subnetworks by folding to form larger networks. Arrays of transputers [3] are employed as the computing engine. The resulting algorithms are found to be superior to conventional solution methods based on ac analysis [4]. Three algorithms using different folding strategies are developed. Results obtained are presented for each algorithm.

EQUIVALENT CIRCUIT FORMULATION

A set of linear algebraic equations can be written if the derivatives of Maxwell's curl equations are replaced by their finite-differene equivalents. These equations represent Kirchoff's equations of a network with network constants made to be functions of the physical medium. Figure 1 shows the network equivalent of a two-dimensional problem in Cartesian coordinates.

The equivalent circuit in fig 1 represents 20 mesh units per guide wavelength at 10 GHz. For large networks the obvious method of nodal admittance representation of the network is uneconomical and inefficient due to matrix inversion requirements. Alternative methods are now applied to solve the problem in a piecewise fashion using diakoptics methods [2,5] that are suitable for concurrent processing.

NETWORK FOLDING

With the diakoptics method a given network is subdivided into a number of subnetworks, each subnetwork is solved independently, then subnetworks are interconnected together. The algorithms presented are based on the circuit analogy suggested by Kron [1] and the fundamental equations of diakoptics proposed by Brameller [5]. Nodal voltages are computed according to:

$$V_a = \tilde{Y}^{-1}_{aa} I_a - \tilde{Y}^{-1}_{aa} C_{a\delta} \tilde{Z}^{-1}_{\delta\delta} C^t_{\delta a} \tilde{Y}^{-1}_{aa} I_a \quad \ldots\ldots \quad (1)$$

where

$\tilde{Z}_{\delta\delta} = Z_{\delta\delta} + C^t_{\delta a} \tilde{Y}^{-1}_{aa} C_{a\delta}$

V_a = Nodal voltages : $\tilde{Y}_{aa}$ = Equivalent NAM

I_a = Excitation currents : $C_{a\delta}$ = Connection matrix

$Z_{\delta\delta}$ = Matrix formed by the removed network

Classical Sequential Diakoptics Method

Using the basic method [5], fig 1 is divided into n subnetworks and solutions of these subnetworks are connected globally according to equation (1). For this we need a maximum of (n+1) matrix inversions, one for each network and the

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(n+1)th due to the removed network. The disadvantage in this method is as the number of subnetworks increases the matrix size due to the removed network $\tilde{Z}_{\delta\delta}$ grows and the method becomes inefficient, also the process of subnetwork interconnection cannot start until the matrix due to the removed network is completed, which in turn requires the subnetwork solutions are completed on a global connection basis.

Concurrent Diakoptics Method

The disadvantages stated above can be overcome by local nearest neighbour subnetwork connection instead of the conventional sequential method of global interconnection. Since, in general diakoptics formulation, the connection of a pair of subnetworks does not depend on that of any other pair, a number of interconnection processes can be implemented in parallel. For instance the subnetworks (1,2),(3,4),(5,6) and (7,8) of fig. 2 can be connected independently. While a speed up is obtained in computing compared with the sequential global method and NAM methods, the disadvantage remains that storage size increases to the point where, as the network size grows, this method requires more CPU time, tending towards that of the above two methods.

Description of Transputer Network Formulation Methods

Figures 2 and 3 show two concurrent network folding strategies. The operation of these algorithms is akin to paper folding with each incremental circuit element forming a hinged square, thus we use the term ORIGAMI algorithm. For this problem concurrency is obtained by executing tasks 1, 2, 3 and 4 simultaneously on different transputers. Transputers [3] are used throughout as a low-cost MIMD parallel computing engine.

In the ORIGAMI algorithms the subnetworks are solved by inverting their admittance matrices. Any two nearest local subnetworks are joined to form a larger subnetwork as soon as the local $\tilde{Z}_{\delta\delta}$ (due to eqn. 1) connecting them becomes available. Such a process can be carried out concurrently as mentioned earlier.

Alternatively these solutions can be obtained directly by growing subnetworks, adding one branch at a time (the SPAWNING algorithm). In the SPAWNING algorithm joining of a new branch creates a new node if its other end is connected to reference node or an already existing node, thus increasing the size of the matrix. On the other hand the matrix size will not change if one end of the branch is connected to an existing node and the other end to reference node or another previously existing node. It is also possible to join two local subnetworks by connecting one branch at a time, so that the matrix inversion due to the removed network can be avoided. This method offers the advantage that any part of a subnetwork can be modified independently allowing opportunities for localised spacial optimization of circuit performance. This method also has computational advantage over the matrix inversion procedure. Fig 2 shows a combined situation with SPAWNING used to generate the subnetworks, that are then compacted using the ORIGAMI algorithm. A more detailed discussion of the SPAWNING algorithms and results obtained are not given here due to space limitations.

RESULTS

For air dielectric the equivalent circuit values of fig. 1 for lossless WR-90 waveguide at 10 GHz are L = 4.10 nH, C = 0.029 pF. The algorithms described above are implemented both in parallel and pipeline versions. Table 1 gives typical execution times obtained on a single T-800 transputer. 3L- parallel Fortran [6] was used throughout. The standing wave pattern obtained by open circuiting the output of the waveguide in the yz- plane is given in fig 4. These results are in good agreement with analytical data [7]. Computation times shown in table 1 for a 96 node problem are found be much superior to those when conventional sequential algorithms such as SPICE [4] are used. Results obtained on an an array of two transputers for a 273 node problem are presented in table 2.

CONCLUSIONS

This paper presented some folding (ORIGAMI) and unfolding (SPAWNING) strategies for concurrent solutions of E.M. field electrical equivalent networks. The methods illustrated are efficient in the mapping of large lumped equivalent circuits onto transputer arrays. The resulting algorithms implemented in ORIGAMI and SPAWNING versions are computationally efficient when compared with their sequential counterparts. The concurrent diakoptics methodologies based on Kron's equivalent network approach would appear to offer advantages for dynamic field modelling when compared with more traditional numerical approaches. This is particularly true when non uniform step size descriptions must be used eg for infinitely spaced boundaries or when non-Cartesian geometries are involved or when localised circuit modifications are required.

REFERENCES

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[6] "Parallel Fortran User Guide", 3L Ltd, Scotland, 1988.

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TABLE 1.

<table>
<tr><td rowspan="3">Sl. No.</td><td rowspan="3">Transputer Type</td><td colspan="5">CPU time in Seconds</td></tr>
<tr><td colspan="2">ORIGAMI Concurrent</td><td colspan="2">ORIGAMI pipeline</td><td>SPAWNING</td></tr>
<tr><td>ver. 1</td><td>ver. 2</td><td>ver. 1</td><td>ver. 2</td><td>ver. 2</td></tr>
<tr><td>1.</td><td>T-800 (@25 MHz)</td><td>10.82</td><td>6.04</td><td>11.37</td><td>6.14</td><td>4.50</td></tr>
</table>

ver. 1: with conventional connection matrix **ver. 2:** with condensed connection matrix
Using **SPICE** on VAX-8650 mainframe : 69.20 seconds

TABLE 2.

<table>
<tr><td rowspan="2">Sl. No.</td><td rowspan="2">Transputer Type</td><td colspan="3">SPAWNING/ORIGAMI Concurrent</td></tr>
<tr><td>CPU Time (Sec.)</td><td>Advantage (Sec.)</td><td>Speedup</td></tr>
<tr><td>1.</td><td>1xT-800 (@25 MHz)</td><td>22.57</td><td rowspan="2">9.72</td><td rowspan="2">1.76</td></tr>
<tr><td>2.</td><td>1xT-800 (@25 MHz)
& 1xT-800 (@20 MHz)</td><td>12.85</td></tr>
</table>

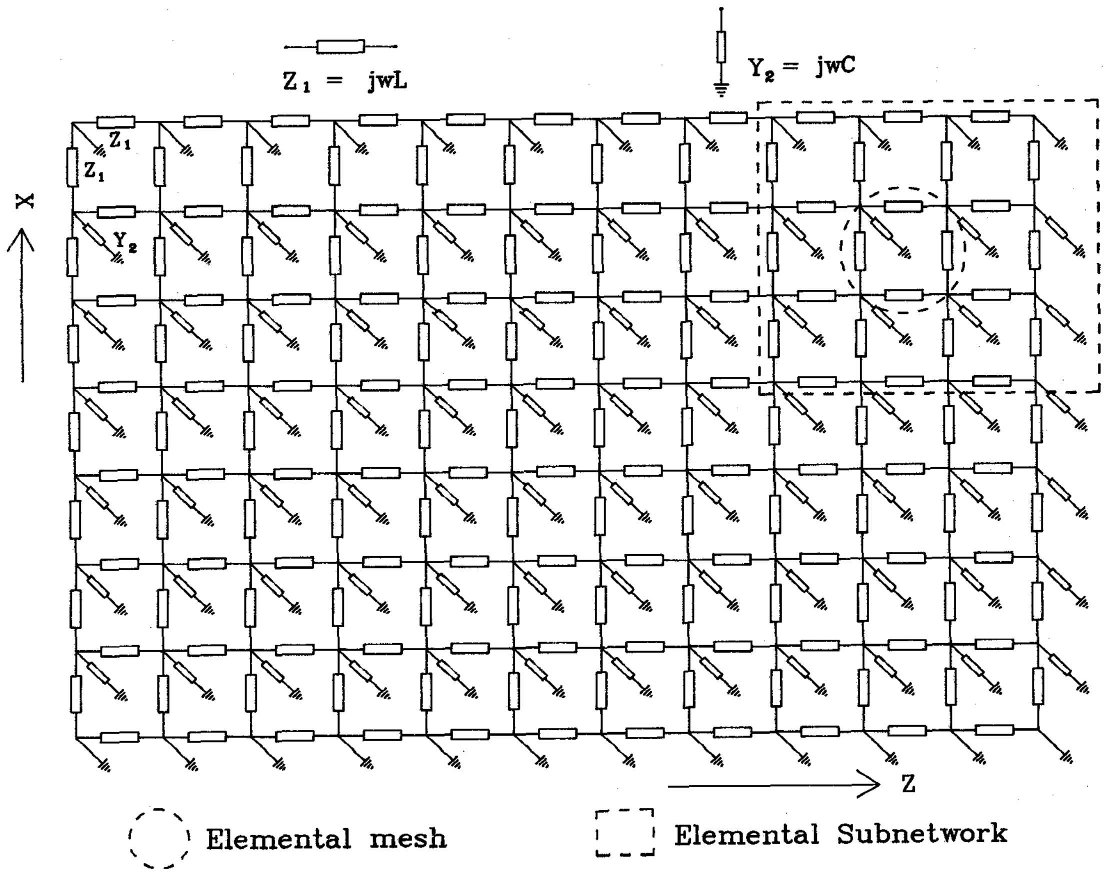

Fig. 1 Equivalent circuit to simulate TE/TM modes in rectangular W/G

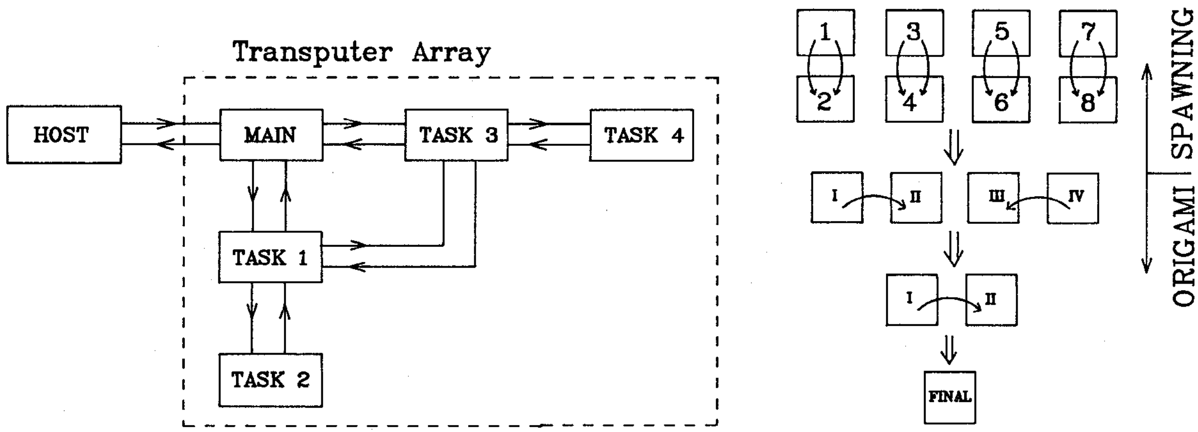

Fig. 2 Concurrent algorithm & Task distribution (Table 1,2)

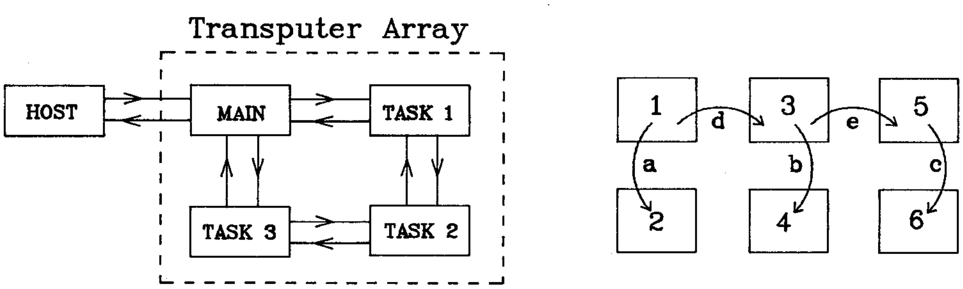

Fig. 3 ORIGAMI Pipline algorithm & Task distribution (Table 1)

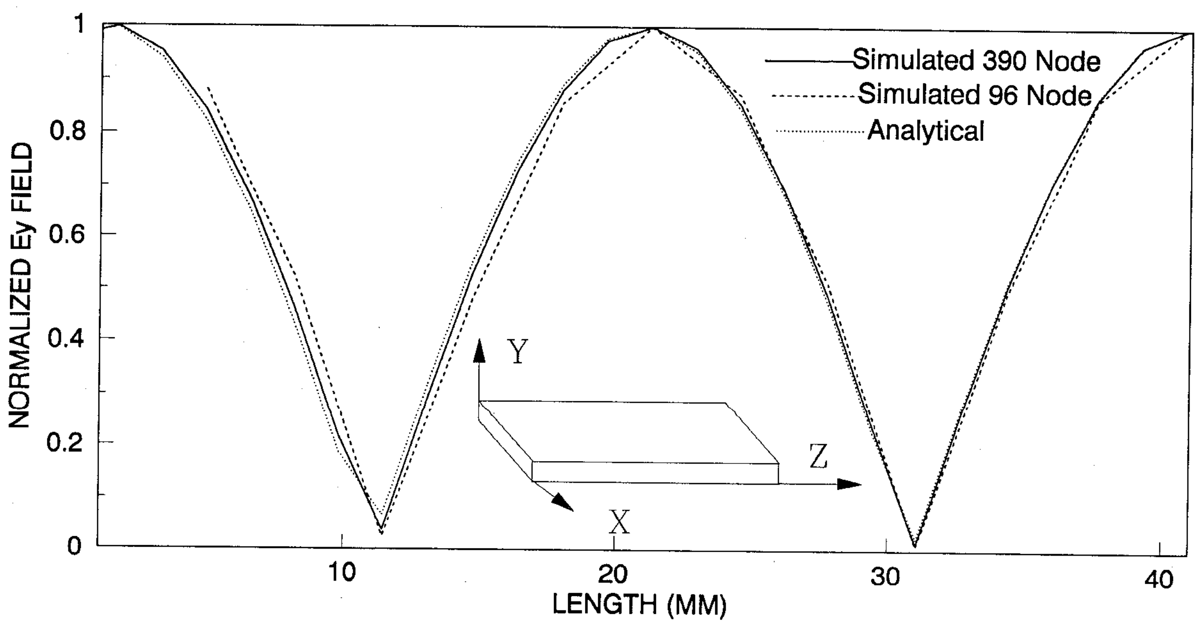

Fig. 4 Ey field in rectangular waveguide in XZ- plane